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New massive supergravity multiplets

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Abstract: We present new off-shell formulations for the massive superspin-3/2 multiplet. In the massless limit, they reduce respectively to the old minimal ($n = -1/3$) and non-minimal ($n \neq -1/3, 0$) linearized formulations for 4D $\mathcal{N} = 1$ supergravity. Duality transformations, which relate the models constructed, are derived.

Keywords: Supergravity Models, Superspaces, Supersymmetric Effective Theories, Supersymmetry and Duality.

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1. Introduction

Four-dimensional $\mathcal{N} = 1$ supergravity exists in several off-shell incarnations. They differ in the structure of their auxiliary fields and, as a consequence, in their matter couplings to supersymmetric matter. It is an ancient tradition[1] to label the off-shell $\mathcal{N} = 1$ supergravity formulations by a parameter n, with its different values corresponding to the following supergravity versions:

i) non-minimal ($n \neq -1/3, 0$) [3, 2, 4];

ii) old minimal ($n = -1/3$) [5, 6];

iii) new minimal ($n = 0$) [7].

Comprehensive reviews of these formulations can be found in [8, 9]. At the linearized level, there also exists a third minimal realization for the massless $(3/2, 2)$ supermultiplet [10], which is reminiscent of the new minimal formulation. The three minimal formulations and the non-minimal series turn out to comprise all possible ways to realize the irreducible massless superspin-3/2 multiplet as a gauge theory of a real axial vector H_a (gravitational superfield) and special compensator(s) [11]. Somewhat unexpectedly, a proliferation of off-shell formulations emerges in the massive case.

[1] It goes back to 1977 when the prepotential formulation for $\mathcal{N} = 1$ superfield supergravity was first developed [1, 2].

On the mass shell, there is a unique way to realize the massive superspin-3/2 multiplet (or massive graviton multiplet) in terms of a real (axial) vector superfield H_a. The corresponding equations [12, 9, 10] are:

$$(\Box - m^2)H_{\alpha\dot\alpha} = 0\,, \quad D^\alpha H_{\alpha\dot\alpha} = 0\,, \quad \overline{D}^{\dot\alpha} H_{\alpha\dot\alpha} = 0 \quad \Longrightarrow \quad \partial^{\alpha\dot\alpha} H_{\alpha\dot\alpha} = 0\,. \tag{1.1}$$

It turns out that no action functional exists to generate these equations if $H_{\alpha\dot\alpha}$ is the only dynamical variable [10]. However, such an action can be constructed if one allows for auxiliary superfields φ with the property that the full mass shell is equivalent to the equations (1.1) together with $\varphi = 0$. Several supersymmetric models with the required properties have been proposed [10, 13, 14]. In particular, for each of the three minimal formulations for linearized supergravity, massive extensions have been derived [13, 14]. By applying superfield duality transformations to these theories, one generates three more models [14] two of which originally appeared in [10].

The present paper continues the research initiated in [10, 13, 14]. We propose new off-shell formulations for the massive superspin-3/2 multiplet. In particular, we derive two new massive extensions of old minimal supergravity, which possess quite interesting properties, as well as a massive extension of non-minimal supergravity.

2. Minimal supergravity multiplets and their massive extensions

In this section, we review the linearized actions for the three minimal supergravity formulations, and recall their massive extensions proposed in [13, 14]. These massive actions possess nontrivial duals [10, 14], which are collected in the appendix.

2.1 Minimal supergravity multiplets

Throughout this paper, we use a reduced set [11] of the superprojectors [15] for the gravitational superfield $H_{\alpha\dot\alpha}$:

$$\Pi^L_0 H_{\alpha\dot\alpha} = -\frac{1}{32}\frac{\partial_{\alpha\dot\alpha}}{\Box^2}\{D^2, \overline{D}^2\}\partial^{\beta\dot\beta}H_{\beta\dot\beta}\,, \tag{2.1}$$

$$\Pi^L_{\frac{1}{2}} H_{\alpha\dot\alpha} = \frac{1}{16}\frac{\partial_{\alpha\dot\alpha}}{\Box^2}D^\gamma\overline{D}^2 D_\gamma \partial^{\beta\dot\beta}H_{\beta\dot\beta}\,, \tag{2.2}$$

$$\Pi^T_{\frac{1}{2}} H_{\alpha\dot\alpha} = \frac{1}{48}\frac{\partial^\beta{}_{\dot\alpha}}{\Box^2}\Big[D_\beta\overline{D}^2 D^\gamma \partial_{(\alpha}{}^{\dot\beta}H_{\gamma)\dot\beta} + D_\alpha\overline{D}^2 D^\gamma\partial_{(\beta}{}^{\dot\beta}H_{\gamma)\dot\beta}\Big]\,, \tag{2.3}$$

$$\Pi^T_1 H_{\alpha\dot\alpha} = \frac{1}{32}\frac{\partial^\beta{}_{\dot\alpha}}{\Box^2}\{D^2, \overline{D}^2\}\partial_{(\alpha}{}^{\dot\beta}H_{\beta)\dot\beta}\,, \tag{2.4}$$

$$\Pi^T_{\frac{3}{2}} H_{\alpha\dot\alpha} = -\frac{1}{48}\frac{\partial^\beta{}_{\dot\alpha}}{\Box^2}D^\gamma\overline{D}^2 D_{(\gamma}\partial_\alpha{}^{\dot\beta}H_{\beta)\dot\beta}\,. \tag{2.5}$$

Here the superscripts L and T denote longitudinal and transverse projectors, while the subscripts $0, 1/2, 1, 3/2$ stand for superspin. Given a local linearized action functional of

$H_{\alpha\dot\alpha}$, it can be expressed in terms of superprojectors using the following identities:

$$D^\gamma \overline{D}^2 D_\gamma H_{\alpha\dot\alpha} = -8\Box\left(\Pi^L_{\frac12} + \Pi^T_{\frac12} + \Pi^T_{\frac32}\right) H_{\alpha\dot\alpha}\,, \tag{2.6}$$

$$\partial_{\alpha\dot\alpha}\partial^{\beta\dot\beta} H_{\beta\dot\beta} = -2\Box(\Pi^L_0 + \Pi^L_{\frac12}) H_{\alpha\dot\alpha}\,, \tag{2.7}$$

$$[D_\alpha, \overline{D}_{\dot\alpha}][D^\beta, \overline{D}^{\dot\beta}] H_{\beta\dot\beta} = 8\Box\left(\Pi^L_0 - 3\,\Pi^T_{\frac12}\right) H_{\alpha\dot\alpha}\,, \tag{2.8}$$

$$\Box H_{\alpha\dot\alpha} = \Box\left(\Pi^L_0 + \Pi^L_{\frac12} + \Pi^T_{\frac12} + \Pi^T_1 + \Pi^T_{\frac32}\right) H_{\alpha\dot\alpha}\,, \tag{2.9}$$

The linearized action for old minimal (type I) supergravity is

$$S^{(\mathrm{I})}[H,\sigma] = \int d^8z\, \Big\{ H^{\alpha\dot\alpha}\Box\Big(\frac12 \Pi^T_{\frac32} - \frac13 \Pi^L_0\Big) H_{\alpha\dot\alpha} - i(\sigma - \overline{\sigma})\partial^{\alpha\dot\alpha} H_{\alpha\dot\alpha} - 3\overline{\sigma}\sigma \Big\}\,. \tag{2.10}$$

Here σ is the chiral compensator, $\overline{D}_{\dot\alpha}\sigma = 0$.

The linearized action for new minimal (type II) supergravity is

$$S^{(\mathrm{II})}[H,\mathcal{U}] = \int d^8z\, \Big\{ H^{\alpha\dot\alpha}\Box\Big(\frac12 \Pi^T_{\frac32} - \Pi^T_{\frac12}\Big) H_{\alpha\dot\alpha} + \frac12 \mathcal{U}[D_\alpha, \overline{D}_{\dot\alpha}] H^{\alpha\dot\alpha} + \frac32 \mathcal{U}^2 \Big\}\,. \tag{2.11}$$

Here $\mathcal{U}$ is the real linear compensator, $\overline{D}^2\mathcal{U} = 0$.

Type III supergravity is known at the linearized level [10] only. The corresponding action is

$$S^{(\mathrm{III})}[H,\mathcal{U}] = \int d^8z\, \Big\{ H^{\alpha\dot\alpha}\Box\Big(\frac12 \Pi^T_{\frac32} + \frac13 \Pi^L_{\frac12}\Big) H_{\alpha\dot\alpha} + \mathcal{U}\partial_{\alpha\dot\alpha} H^{\alpha\dot\alpha} + \frac32 \mathcal{U}^2 \Big\}\,. \tag{2.12}$$

Similarly to (2.11), here $\mathcal{U}$ the real linear compensator, $\overline{D}^2\mathcal{U} = 0$.

2.2 Massive extensions

As demonstrated in [13, 14], consistent massive extensions of the supersymmetric theories (2.10), (2.11) and (2.12) can be obtained simply by adding mass terms for the gravitational superfield and for a gauge potential associated with the compensator, with the latter being treated as a gauge-invariant field strength.

Consider first the off-shell massive supergravity multiplet derived in [13]. The chirality constraint on the compensator σ in (2.10) can always be solved in terms of an unconstrained real superfield [16]:

$$\sigma = -\frac14 \overline{D}^2 P\,, \qquad \overline{\sigma} = -\frac14 D^2 P\,, \qquad \overline{P} = P\,. \tag{2.13}$$

Then, the massive extension of (2.10), which was proposed in [13], is

$$S^{(\mathrm{I})}_{\mathrm{mass}}[H,P] = S^{(\mathrm{I})}[H,\sigma] - \frac12 m^2 \int d^8z\, \Big\{ H^{\alpha\dot\alpha} H_{\alpha\dot\alpha} - \frac92 P^2 \Big\}\,. \tag{2.14}$$

The supergravity formulations (2.11) and (2.12) involve the real linear compensator $\mathcal{U}$. The constraint on $\mathcal{U}$ can be solved as follows [17]:

$$\mathcal{U} = D^\alpha \chi_\alpha + \overline{D}_{\dot\alpha}\overline{\chi}^{\dot\alpha}\,, \qquad \overline{D}_{\dot\alpha}\chi_\alpha = 0\,,$$

with χ_α an unconstrained chiral spinor. Adopting χ_α and $\bar\chi_{\dot\alpha}$ as independent dynamical variables to describe the compensator, the new minimal model (2.11) possesses the massive extension [14]

$$S^{(\mathrm{II})}_{\mathrm{mass}}[H,\chi] = S^{(\mathrm{II})}[H,\mathcal{U}] - \frac{1}{2}m^2 \int d^8z\, H^{\alpha\dot\alpha}H_{\alpha\dot\alpha} + 3m^2\Big\{\int d^6z\,\chi^2 + \mathrm{c.c.}\Big\}\ . \quad (2.15)$$

Similarly, the type III model (2.12) possesses the following massive extension [14]

$$S^{(\mathrm{III})}_{\mathrm{mass}}[H,\chi] = S^{(\mathrm{III})}[H,\mathcal{U}] - \frac{1}{2}m^2 \int d^8z\, H^{\alpha\dot\alpha}H_{\alpha\dot\alpha} - 9m^2\Big\{\int d^6z\,\chi^2 + \mathrm{c.c.}\Big\}\ . \quad (2.16)$$

3. New massive supergravity multiplets

In the previous section we have reviewed several known formulations for the massive superspin-3/2 multiplet. They constitute massive extensions of the minimal supergravity formulations with $12+12$ off-shell degrees of freedom. Now, we are going to obtain a massive extension of the non-minimal supergravity formulation with 20+20 off-shell degrees of freedom. In the notation of [9], the linearized action for non-minimal supergravity [2] is as follows:

$$\begin{aligned}
S^{\mathrm{NM}}[H,\Sigma] = \int d^8z\Big[&-\frac{1}{16}H^{\alpha\dot\alpha}D^\beta\overline{D}^2D_\beta H_{\alpha\dot\alpha} + \frac{n+1}{8n}(\partial_{\alpha\dot\alpha}H^{\alpha\dot\alpha})^2 \\
&+\frac{n+1}{32}([D_\alpha,\overline{D}_{\dot\alpha}]H^{\alpha\dot\alpha})^2 - \frac{(n+1)(3n+1)}{4n}iH^{\alpha\dot\alpha}\partial_{\alpha\dot\alpha}(\Sigma-\overline\Sigma) \\
&-\frac{3n+1}{4}H^{\alpha\dot\alpha}(D_\alpha\overline{D}_{\dot\alpha}\Sigma - \overline{D}_{\dot\alpha}D_\alpha\overline\Sigma) \\
&+\frac{(3n+1)^2}{4n}\overline\Sigma\Sigma + \frac{9n^2-1}{8n}(\Sigma^2+\overline\Sigma^2)\Big] \\
= \int d^8z\Big[&\frac{1}{2}H^{\alpha\dot\alpha}\Box\Big(\Pi^T_{\frac{3}{2}} + \frac{(n+1)^2}{2n}\Pi^L_0 + \frac{3n+1}{2n}\Pi^L_{\frac{1}{2}} - \frac{3n+1}{2}\Pi^T_{\frac{1}{2}}\Big)H_{\alpha\dot\alpha} \\
&-\frac{(n+1)(3n+1)}{4n}iH^{\alpha\dot\alpha}\partial_{\alpha\dot\alpha}(\Sigma-\overline\Sigma) + \frac{(3n+1)^2}{4n}\overline\Sigma\Sigma \\
&-\frac{3n+1}{4}H^{\alpha\dot\alpha}(D_\alpha\overline{D}_{\dot\alpha}\Sigma - \overline{D}_{\dot\alpha}D_\alpha\overline\Sigma) + \frac{9n^2-1}{8n}(\Sigma^2+\overline\Sigma^2)\Big]\ ,
\end{aligned} \quad (3.1)$$

Here $n \neq -1/3, 0$, and the compensator Σ is a complex linear superfield obeying the only constraint $\overline{D}^2\Sigma = 0$. For simplicity, the parameter n is chosen in (3.1) to be real, see [2, 4] for the general case of complex n.

From the point of view of massive supergravity, the non-minimal formulation appears to be quite special. It turns out that there is no consistent massive extension of the theory (3.1) obtained by adding mass terms for the gravitational superfield and for the gauge spinor potential Ψ_α associated with the non-minimal compensator $\overline\Sigma = D^\alpha\Psi_\alpha$.[2]

[2]Note that, of course, the unconstrained superfield prepotential superfield Ψ^α is not chiral, unlike χ_α in the new minimal case.

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This fact is in obvious contrast with the minimal supergravity formulations discussed in the previous section. We will come back to a discussion of these points in section 4.

3.1 Massive extensions of old minimal supergravity

To derive a massive extension of (3.1), one can try to employ the idea that the old minimal and non-minimal supergravity formulations are dually equivalent, see e.g. [8, 9] for reviews. In order to apply duality considerations in the massive case, however, it is necessary to have an appropriate massive extension of the action (2.10). It turns out that the formulation (2.14) is not well suited.

Therefore, as a first step, let us actually derive a new massive extension of the old minimal supergravity formulation (2.10). As compared with (2.14), such an extension appears to be more natural, for the chiral compensator is defined through an *unconstrained complex* superfield F:

$$\sigma = -\frac{1}{4}\overline{D}^2 F\,, \qquad \overline{\sigma} = -\frac{1}{4}D^2\overline{F}\,. \tag{3.2}$$

We choose the simplest ansatz for the massive action:

$$\tilde{S}^{(\mathrm{I})}_{\mathrm{mass}}[H,F] = S^{(\mathrm{I})}[H,\sigma] - m^2\int d^8z\,\Big\{\frac{1}{2}H^{\alpha\dot\alpha}H_{\alpha\dot\alpha} - aF\overline{F}\Big\}\,, \tag{3.3}$$

with $a \neq 0$ a real constant.

To prove that (3.3) indeed describes a massive superspin-3/2 multiplet, for a special value of the parameter a, we study the corresponding equations of motion:

$$0 = \Box\Big(-\frac{2}{3}\Pi^L_0 + \Pi^T_{\frac{3}{2}}\Big)H_{\alpha\dot\alpha} + i\partial_{\alpha\dot\alpha}(\sigma-\overline{\sigma}) - m^2 H_{\alpha\dot\alpha}\,, \tag{3.4}$$

$$0 = \frac{i}{4}\overline{D}^2\partial^{\alpha\dot\alpha}H_{\alpha\dot\alpha} - \frac{3}{16}\overline{D}^2D^2\overline{F} + a\,m^2\overline{F}\,, \tag{3.5}$$

$$0 = -\frac{i}{4}D^2\partial^{\alpha\dot\alpha}H_{\alpha\dot\alpha} - \frac{3}{16}D^2\overline{D}^2F + a\,m^2F\,. \tag{3.6}$$

Since $a \neq 0$ and $m \neq 0$, the equations (3.5) and (3.6) imply $\overline{D}_{\dot\alpha}\overline{F} = D_\alpha F = 0$. Now, we can use the identity $\frac{1}{16}D^2\overline{D}^2 + \frac{1}{16}\overline{D}^2D^2 + \frac{1}{8}D_\alpha\overline{D}^2D^\alpha = \Box$, in order to rewrite eqs. (3.5) and (3.6) as

$$0 = \frac{i}{4}\overline{D}^2\partial^{\alpha\dot\alpha}H_{\alpha\dot\alpha} - 3\Box\overline{F} + a\,m^2\overline{F}\,, \tag{3.7}$$

$$0 = -\frac{i}{4}D^2\partial^{\alpha\dot\alpha}H_{\alpha\dot\alpha} - 3\Box F + a\,m^2F\,. \tag{3.8}$$

Next, by applying $\frac{i}{4}\overline{D}^2\partial^{\alpha\dot\alpha}$ to eq. (3.4) and making use of eq. (3.7), we arrive at

$$\Box\Big(\frac{2a}{3} - 3\Big)\overline{F} + a\,m^2\overline{F} = 0\,. \tag{3.9}$$

Choosing $a \equiv \frac{9}{2}$ gives $\overline{F} = 0 = F$ on the mass shell. After that, eqs. (3.7) and (3.8) give $\Pi^L_0 H_{\alpha\dot\alpha} = 0$. Finally, by applying to equation (3.4) respectively the projectors $\Pi^L_{\frac{1}{2}}$, $\Pi^T_{\frac{1}{2}}$ and Π^T_1 we find

$$\Pi^L_{\frac{1}{2}}H_{\alpha\dot\alpha} = \Pi^T_{\frac{1}{2}}H_{\alpha\dot\alpha} = \Pi^T_1 H_{\alpha\dot\alpha} = 0\,. \tag{3.10}$$

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The only non-zero projected component is $\Pi^T_{\frac{3}{2}} H_{\alpha\dot\alpha}$ which is now equal to $H_{\alpha\dot\alpha}$ and, once simplified equation (3.4), results to satisfy the Klein-Gordon equation. All the previous relations imply that on-shell $H_{\alpha\dot\alpha}$, satisfy equations (1.1), and it describes the irreducible massive superspin-3/2 multiplet. The final action is:

$$\tilde{S}^{(\mathrm{I})}_{\mathrm{mass}}[H,F] = S^{(\mathrm{I})}[H,\sigma] - \frac{1}{2}m^2 \int d^8z \left\{ H^{\alpha\dot\alpha} H_{\alpha\dot\alpha} - 9F\overline{F} \right\}, \tag{3.11}$$

with σ expressed via F according to (3.2).

The model constructed, eq. (3.11), can be related to that given in (2.14). Indeed, let us consider the following nonlocal field redefinition (compare with [9]):

$$F = -\frac{1}{4}\frac{D^2}{\Box}\sigma + \varphi + \frac{1}{\sqrt{2}}(\mathcal{U} + \mathrm{i}\,\mathcal{V}) \ . \tag{3.12}$$

Here σ and φ are chiral scalars, $\overline{D}_{\dot\alpha}\sigma = \overline{D}_{\dot\alpha}\varphi = 0$, while $\mathcal{U}$ and $\mathcal{V}$ are real linear superfields,

$$\overline{D}^2\mathcal{U} = \overline{D}^2\mathcal{V} = 0\,, \qquad \overline{\mathcal{U}} = \mathcal{U}\,, \qquad \overline{\mathcal{V}} = \mathcal{V} \ . \tag{3.13}$$

We then have

$$\int d^8z\, F\overline{F} = \frac{1}{2}\int d^8z\,(P^2 + V^2)\,, \tag{3.14}$$

where

$$P = -\frac{1}{4}\frac{D^2}{\Box}\sigma - \frac{1}{4}\frac{\overline{D}^2}{\Box}\overline{\sigma} + \mathcal{U}\,, \qquad V = \varphi + \overline{\varphi} + \mathcal{V} \tag{3.15}$$

are unconstrained real superfields. It is obvious that $\sigma = -\frac{1}{4}\overline{D}^2 F = -\frac{1}{4}\overline{D}^2 P$. Let us implement the field redefinition (3.12) in the action (3.11). This gives

$$\tilde{S}^{(\mathrm{I})}_{\mathrm{mass}}[H,F] = S^{(\mathrm{I})}_{\mathrm{mass}}[H,P] + \frac{9}{4}m^2 \int d^8z\, V^2 \ . \tag{3.16}$$

Since V is unconstrained and appears in the action without derivatives, it can be integrated out. This amounts to setting to zero the second term in (3.16).

It is worth saying a few more words about the two solutions, eqs. (2.13) and (3.2), to the chirality constraint in terms of unconstrained superfields. Parametrization (3.2) for the chiral compensator is known to lead to the standard auxiliary fields of minimal supergravity (S, P, A_a). If one instead parametrizes σ according to (2.13), the set of auxiliary fields becomes (S, C_{abc}, A_a). This set includes a gauge three-form C_{abc}, instead of the scalar P. The latter actually occurs as a gauge-invariant field strength associated with C_{abc}. It perhaps is worth noting that this three-form in four dimensions though non-dynamical may be regarded as the truncation of the well-known similar dynamical field that occurs in 11D supergravity and M-theory.

The theory with action (3.11) can be used to construct a dual formulation, in a manner similar to the approach advocated in [14]. Instead of imposing eq. (3.2) as a kinematic constraint, one can generate it as an equation of motion by means of the introduction of an unconstrained complex Lagrange multiplier Y. Consider the following auxiliary action:

$$S = S^{(\mathrm{I})}[H,\sigma] + \int d^8z \Big[-\frac{1}{2}m^2 H^{\alpha\dot\alpha}H_{\alpha\dot\alpha} + \frac{9}{2}m^2 F\overline{F} \\ +3m\,Y\Big(\frac{1}{4}\overline{D}^2 F + \sigma\Big) + 3m\,\overline{Y}\Big(\frac{1}{4}D^2\overline{F} + \overline{\sigma}\Big)\Big]\,. \qquad (3.17)$$

Here σ is a chiral superfield unrelated to F. Varying Y and $\overline{Y}$ enforces the constraints (3.2), and then we are clearly back to (3.11). On the other hand, if we integrate out F and $\overline{F}$ using their equations of motions

$$\frac{3}{2}mF + \frac{1}{4}D^2\overline{Y} = 0\,, \qquad \frac{3}{2}m\overline{F} + \frac{1}{4}\overline{D}^2 Y = 0\,, \qquad (3.18)$$

and introduce the chiral superfield $\chi = -\frac{1}{4}\overline{D}^2 Y$ and its conjugate $\overline{\chi} = -\frac{1}{4}D^2\overline{Y}$, we arrive at the following dual action

$$S = \int d^8z \Big[H^{\alpha\dot\alpha}\Box\Big(\frac{1}{2}\Pi^T_{\frac{3}{2}} - \frac{1}{3}\Pi^L_0\Big)H_{\alpha\dot\alpha} - i(\sigma-\overline{\sigma})\partial^{\alpha\dot\alpha}H_{\alpha\dot\alpha} - 3\sigma\overline{\sigma} \\ -\frac{1}{2}m^2 H^{\alpha\dot\alpha}H_{\alpha\dot\alpha} - 2\overline{\chi}\chi\Big] + 3m\int d^6z\,\chi\,\sigma + 3m\int d^6\bar{z}\,\overline{\chi}\,\overline{\sigma}\,. \qquad (3.19)$$

This dynamical system is quite interesting in its own rights. Unlike the massive models (2.14) and (3.11), the above formulation involves only the chiral compensator of old minimal supergravity, and not its gauge potential. The mass generation becomes possible due to the presence of a second chiral superfield. In a sense, one can also interpret (3.19) as a coupling of the gravitational superfield to a massive $\mathcal{N}=2$ hypermultiplet.

The explicit structure of action (3.19) explains why all attempts have failed to construct a Lagrangian formulation for the massive superspin-3/2 multiplet solely in terms of the old minimal supergravity prepotentials H_a, σ and $\overline{\sigma}$.

3.2 Massive extension of non-minimal supergravity

Up to now we have considered massive extensions of old minimal and new minimal supergravity. Here we would like to address the problem of deriving a massive extension of linearized non-minimal supergravity [2, 4, 8, 9]. This goal can be achieved by performing a different duality transformation starting from (3.11).

In the action (3.17), the superfield σ is chiral by construction. Enforcing the equation of motion for Y constrains F to be related to σ according to (3.2). Clearly, in analogy with the action (A.1), we can actually remove the chirality constraint imposed on σ and choose this superfield to be unconstrained complex off the mass shell. Note also that, in such a setting, we can write a more general action that should reduce to (3.17) once σ is constrained to be chiral, namely[3]

$$S = \int d^8z \Bigg[H^{\alpha\dot\alpha}\Box\Big(\frac{1}{2}\Pi^T_{\frac{3}{2}} - \frac{1}{3}\Pi^L_0\Big)H_{\alpha\dot\alpha}$$
$$+(2a-1)(S-\overline{S})i\partial^{\alpha\dot\alpha}H_{\alpha\dot\alpha} + aS\,D^\alpha\overline{D}^{\dot\alpha}H_{\alpha\dot\alpha} - a\overline{S}\,\overline{D}^{\dot\alpha}D^\alpha H_{\alpha\dot\alpha}$$
$$-3S\overline{S} - b\,\frac{3}{2}\,(S^2+\overline{S}^2) - \frac{1}{2}m^2H^{\alpha\dot\alpha}H_{\alpha\dot\alpha} + \frac{9}{2}m^2F\overline{F}$$
$$+3m\,Y\Big(\frac{1}{4}\overline{D}^2F + S\Big) + 3m\,\overline{Y}\Big(\frac{1}{4}D^2\overline{F}+\overline{S}\Big)\Bigg]\,. \tag{3.20}$$

Clearly, varying Y and $\overline{Y}$ gives $S = \sigma = -\frac{1}{4}\overline{D}^2F$ and the conjugate relation, and then we are still back to (3.3). Instead if we integrate out S, F and their conjugates, using their equations of motion

$$F = -\frac{1}{6m}D^2\overline{Y}\,, \qquad 0 = -S - b\overline{S} - \frac{(2a-1)}{3}i\partial_{\alpha\dot\alpha}H^{\alpha\dot\alpha} - \frac{a}{3}\overline{D}_{\dot\alpha}D_\alpha H^{\alpha\dot\alpha} + m\overline{Y}, \tag{3.21}$$

which imply

$$S = \frac{a}{6(b+1)}[D_\alpha,\overline{D}_{\dot\alpha}]H^{\alpha\dot\alpha} + \frac{(a-1)}{3(b-1)}i\partial_{\alpha\dot\alpha}H^{\alpha\dot\alpha} + \frac{m}{b^2-1}(b\,Y - \overline{Y})\,, \tag{3.22}$$

we arrive at the following action (defining $\Sigma = mY$ and $\chi = -\frac{1}{4}\overline{D}^2Y$):

$$S = \int d^8z\Bigg\{ \frac{1}{2}\,H^{\alpha\dot\alpha}\Box\Bigg[\Pi^T_{\frac{3}{2}} + \Big(\frac{4a^2}{3(b+1)} - \frac{4(a-1)^2}{3(b-1)} - \frac{2}{3}\Big)\Pi^L_0$$
$$-\frac{4(a-1)^2}{3(b-1)}\,\Pi^L_{\frac{1}{2}} - \frac{4a^2}{b+1}\,\Pi^T_{\frac{1}{2}}\Bigg]H_{\alpha\dot\alpha}$$
$$-\frac{1}{2}m^2H^{\alpha\dot\alpha}H_{\alpha\dot\alpha} + \Big(\frac{2a-b-1}{b^2-1}\Big)(\Sigma-\overline{\Sigma})i\partial^{\alpha\dot\alpha}H_{\alpha\dot\alpha}$$
$$+\frac{a}{b+1}\,H^{\alpha\dot\alpha}(D_\alpha\overline{D}_{\dot\alpha}\Sigma - \overline{D}_{\dot\alpha}D_\alpha\overline{\Sigma}) + \frac{3b}{2(b^2-1)}(\Sigma^2+\overline{\Sigma}^2)$$
$$-\frac{3}{b^2-1}\,\Sigma\overline{\Sigma} - 2\chi\overline{\chi}\Bigg\}\,, \tag{3.23}$$

with the dynamical variables χ and Σ constrained as follows:

$$\overline{D}_{\dot\alpha}\chi \,=\, 0\,, \qquad -\frac{1}{4}\overline{D}^2\Sigma \,=\, m\,\chi\,. \tag{3.24}$$

These constraints describe a chiral-non-minimal (CNM) doublet [18] (see also [19] for recent results on the quantum beahviour of CNM multiplets). We have thus constructed a CNM formulation for massive supergravity. In particular, it is easy to see that the choice

$$a = -\frac{1}{2}\,, \qquad b = -\frac{3n-1}{3n+1} \tag{3.25}$$

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[3] One can actually consider even more general action by letting the parameter a and b to be complex, $a(S\,D^\alpha\overline{D}^{\dot\alpha}H_{\alpha\dot\alpha} - \overline{S}\,\overline{D}^{\dot\alpha}D^\alpha H_{\alpha\dot\alpha}) \;\rightarrow\; (aS\,D^\alpha\overline{D}^{\dot\alpha}H_{\alpha\dot\alpha} - \overline{a}\overline{S}\,\overline{D}^{\dot\alpha}D^\alpha H_{\alpha\dot\alpha})$ and $b(S^2+\overline{S}^2) \;\rightarrow\; (bS^2 + \overline{b}\,\bar{S}^2)$. For simplicity, we restrict our consideration to the case $a = \overline{a}$ and $b = \overline{b}$.

corresponds to

$$
\begin{aligned}
S^{\rm NM}_{\rm mass} = \int d^8z \Big[&-\frac{1}{16}\,H^{\alpha\dot\alpha}D^\beta\overline{D}^2D_\beta H_{\alpha\dot\alpha}+\frac{n+1}{8n}\,(\partial_{\alpha\dot\alpha}H^{\alpha\dot\alpha})^2 \\
&+\frac{n+1}{32}\,([D_\alpha,\overline{D}_{\dot\alpha}]H^{\alpha\dot\alpha})^2-\frac{(n+1)(3n+1)}{4n}\,iH^{\alpha\dot\alpha}\partial_{\alpha\dot\alpha}(\Sigma-\overline{\Sigma}) \\
&-\frac{3n+1}{4}\,H^{\alpha\dot\alpha}(D_\alpha\overline{D}_{\dot\alpha}\Sigma-\overline{D}_{\dot\alpha}D_\alpha\overline{\Sigma})+\frac{(3n+1)^2}{4n}\,\overline{\Sigma}\Sigma \\
&+\frac{9n^2-1}{8n}\,(\Sigma^2+\overline{\Sigma}^2)-\frac{1}{2}m^2H^{\alpha\dot\alpha}H_{\alpha\dot\alpha}-2\,\overline{\chi}\chi\Big]\,, \qquad (3.26)
\end{aligned}
$$

with χ and Σ constrained as in (3.24). This model can be recognized to be the desired massive extension of non-minimal supergravity (3.1). Since we have derived (3.26) by applying a superfield duality transformation to (3.11), the two theories are equivalent and describe the massive superspin-3/2 multiplet.

One can also obtain the non-minimal formulation (3.26), (3.24) using a slightly different path. The linearized supergravity actions (2.10) and (3.1) are known to be dual to each other. The duality proceeds, say, by making use of the auxiliary action

$$
S[H,\Sigma,\sigma] = S^{\rm NM}[H,\Sigma]-3\int d^8z\Big[\,\sigma\Sigma+\overline{\sigma}\overline{\Sigma}\,\Big]\,, \qquad (3.27)
$$

where σ is chiral and Σ is unconstrained. Varying σ in (3.27) makes Σ linear and then we are back to linearized non-minimal supergravity described by (3.1). Instead, integrating out Σ and $\overline{\Sigma}$ leads to the old minimal supergravity action (2.14).

Now, in order to find a massive extension of (3.1), we can start directly from the above action (3.27) extended in the following way

$$
\begin{aligned}
S_m[H,\Sigma,\sigma,\chi] = S^{\rm NM}[H,\Sigma]&+\int d^8z\Big[-3(\sigma\Sigma+\overline{\sigma}\overline{\Sigma})-\frac{1}{2}m^2H^{\alpha\dot\alpha}H_{\alpha\dot\alpha}-2\,\overline{\chi}\chi\Big] \\
&+3m\int d^6z\;\chi\,\sigma+3m\int d^6\bar{z}\;\overline{\chi}\,\overline{\sigma}\;\;, \qquad (3.28)
\end{aligned}
$$

where χ is chiral. Integrating out Σ and $\overline{\Sigma}$, we arrive at the action (3.19) which is known to be dual to linearized old minimal supergravity (2.14). Instead, integrating out σ and $\overline{\sigma}$ leads to the massive non-minimal formulation (3.26), (3.24).

Let us analyse the compensator sector of (3.26) which is obtained by setting $H_a=0$. Up to a sign, it corresponds to a massive chiral-non-minimal (CNM) multiplet [18]. Such a multiplet can be viewed as the mechanism to generate a mass for the complex linear superfield Σ in the presence of a chiral superfield χ by means of a consistent deformation of the off-shell constraint: $\overline{D}^2\Sigma=0\rightarrow\overline{D}^2\Sigma=-4m\chi$. The CNM multiplet is known to be dual to a pair of chiral superfields having a Dirac mass term of the form $(m\int d^6z\,\sigma\chi+\text{c.c.})$; this multiplet is sometimes called chiral-chiral (CC). The compensator sector of (3.19) is clearly described by a CC multiplet. It is worth pointing out that CNM multiplets are ubiquitous in $\mathcal{N}=2$ supersymmetry in the framework of projective superspace; see [20–23] for references on 4D projective superspace and also [24] for extensions to 5 and 6 dimensions.

We close the section by observing that in the CC and CNM massive supergravity formulations developed, see eqs. (3.19) and (3.23)–(3.26) respectively, the mass parameter m can be easily promoted to become complex. This is different from the previously known formulation described in the appendix, and could be a relevant property when trying to extend these multiplets to extra-dimensions in particular for the $D > 5$ case.

4. Discussion

In this work, we have continued (and hopefully completed) a program of the exploration of the structure of massive linearized 4D $\mathcal{N} = 1$ superfield supergravity models. One of the points of this continued effort is to establish a number of benchmarks for other purposes.

First, it is known that closed superstring theories and M-theory, when truncated to four dimensions, must possess massive spin-2 (and higher) multiplets in a low-energy effective action. Thus our effort is part of the long-term program begun in references [10, 11] to gain a systematic understanding first of the massive superspin-3/2 system and later all of arbitrary 4D $\mathcal{N} = 1$ higher spin multiplets.

Second, massive theories are also interesting to study as a step toward the realization of higher values of D as shown in the work of [25, 26]. There a successful approach was given in the case of 5D supergravity. However, to date no successful extension of this construction is known for higher values of D. Thus, this present effort also is a probe for furthering this program of constructing (at least) linearized versions of all higher D supergravity theories in terms of 4D, $\mathcal{N} = 1$ superfields.

We have presented the first successful description of the massive version of linearized non-minimal 4D $\mathcal{N} = 1$ superfield supergravity. As well we have obtained results that show signs of $\mathcal{N} = 2$ supermultiplet being very relevant to this course of study. This result is important in a way that may also open a new view of the five-dimensional theory. The version of the 5D theory constructed in [25] only possesses 5D Lorentz invariance *on-shell*. This is manifest in the fact that though the physical spinors in the work by Linch, Luty and Phillips [25] are proper 5D spinors, the auxiliary spinors in the work are not. The supergravity multiplet in this work is described by the old minimal supergravity theory given in [16]. This possesses no *auxiliary* spinors. A distinguishing point of our present work is that by describing the supergravity multiplet in terms of non-minimal supergravity, there opens the possibility to contruct a 5D extension where the auxiliary spinors also describe *off-shell* 5D spinors.

Conceptually, the structure of the massive non-minimal action (3.26) differs considerably from the massive minimal models (2.14), (2.15) and (2.16), in the sense that (3.26) does not involve any mass term for the gauge spinor potential Ψ_α associated with the non-minimal compensator $\overline{\Sigma} = D^\alpha \Psi_\alpha$. To explain this feature, let us consider the massive extensions of old minimal supergravity (2.14), (3.11) and (3.19). These three actions look identical in the sector involving the gravitational superfield. Their parts involving the compensators only, obtained by setting $H_a = 0$, look quite different. Nevertheless, they all share one important common feature: on the mass shell, they describe two free massive superspin-0 multiplets. The same property holds for the compensator sector of the non-minimal action (3.26). That is, it describes a free massive $\mathcal{N}=2$ hypermultiplet, or two free massive $\mathcal{N}=1$ superspin-0 multiplets. Let us now introduce a massive extension for the compensator part of (3.1). This is as follows [9]:

$$S = -\int d^8 z \Big[\Sigma\,\overline{\Sigma} + \frac{\zeta}{2}(\Sigma^2 + \overline{\Sigma}^2) + 2m(\Psi^\alpha \Psi_\alpha + \overline{\Psi}_{\dot\alpha}\overline{\Psi}^{\dot\alpha})\Big] , \tag{4.1}$$

with ζ a parameter. Unlike the compensator sector of (3.26), this action describes a single superspin-0 multiplet, since the equations of motion imply

$$-\frac{1}{4}D^2\Sigma + m\overline{\Sigma} = 0 \ . \tag{4.2}$$

As a result, the action (4.1) can not be used for generating a massive extension of linearized non-minimal supergravity. It is worth pointing out that in the massless case, the parameter ζ can take arbitrary values except ± 1 [18]. In the massive case, no restriction on ζ occurs, since the corresponding term in (4.1) can be completely removed by a field redefinition $\Psi_\alpha \to \Psi_\alpha + (\lambda/m) D^2 \Psi_\alpha$, with λ a parameter.

To conclude this paper, we would like to comment upon a subtle property of the massless action (3.1) in respect to the classification of linearized supergravity models given in [11]. The linearized action for non-minimal supergravity is defined for $n \neq -1/3, 0$. Looking at the second form for the action (3.1), in terms of the superprojectors, one clearly sees that the case $n=-1$ is very special. In this and only this case, the action involves only three superprojectors. The latter feature appears to be in a seeming contradiction with the theorem in [11] that there are no irreducible supergravity multiplets with three superprojectors in the action. Fortunately, this contradiction can be readily resolved if one recalls the structure of the linearized gauge transformations in non-minimal supergravity [9]:

$$\begin{aligned} \delta H_{\alpha\dot\alpha} &= \overline{D}_{\dot\alpha} L_\alpha - D_\alpha \overline{L}_{\dot\alpha} \,, \\ \delta\Sigma &= -\frac{1}{4}\frac{n+1}{3n+1}\,\overline{D}^2 D^\alpha L_\alpha - \frac{1}{4}\overline{D}_{\dot\alpha} D^2 \overline{L}^{\dot\alpha} \,, \end{aligned} \tag{4.3}$$

with L_α an unconstrained gauge parameter. As may be seen, the gauge freedom allows one to completely gauge away the complex linear compensator Σ provided $n \neq -1$. This is no longer true for $n \neq -1$ (in which case the compensator can be gauged away on the mass shell only).[4] On the other hand, the classification given in [11] applies to those off-shell realizations for the massless superspin-3/2 multiplet, which can be formulated solely in terms of the gravitational superfield upon gauging away the compensator(s).

We hope that the present work has brought the topic of massive off-shell superspin-3/2 multiplets to the same level of completeness as that existing for the massive gravitino multiplets [14, 28].

[4] This property of $n=-1$ supergravity is generic within the so-called gauge transversal formulation for massless multiplets of half-integer superspin $Y \geq 3/2$ [27]. The transversal series terminates at $Y = 3/2$ at the $n=-1$ formulation for non-minimal supergravity.

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A. Dual actions

In this appendix, we collect the dual formulations for the massive minimal models given subsection 2.2 following [14].

The theory with action $S^{(\mathrm{I})}_{\mathrm{mass}}[H,P]$, eq, (2.14), possesses a dual formulation. Let us introduce the "first-order" action

$$S_{\mathrm{Aux}} = \int d^8z \Big\{ H^{\alpha\dot\alpha}\Box\Big(\frac{1}{2}\Pi^T_{\frac{3}{2}} - \frac{1}{3}\Pi^L_0\Big)H_{\alpha\dot\alpha} - \frac{1}{2}m^2 H^{\alpha\dot\alpha}H_{\alpha\dot\alpha} - U\partial^{\alpha\dot\alpha}H_{\alpha\dot\alpha} \\ -\frac{3}{2}U^2 + \frac{9}{4}m^2P^2 + 3mV\Big(U + \frac{i}{4}\overline{D}^2P - \frac{i}{4}D^2P\Big)\Big\}, \tag{A.1}$$

where U and V are real unconstrained superfields. Varying V brings us back to (2.14). On the other hand, we can eliminate U and P using their equations of motion. With the aid of (2.7), this gives

$$S^{(\mathrm{IB})}[H,P] = \int d^8z \Big\{ H^{\alpha\dot\alpha}\Box\Big(\frac{1}{2}\Pi^T_{\frac{3}{2}} + \frac{1}{3}\Pi^L_{\frac{1}{2}}\Big)H_{\alpha\dot\alpha} - \frac{1}{2}m^2H^{\alpha\dot\alpha}H_{\alpha\dot\alpha} \\ -\frac{1}{16}V\{\overline{D}^2, D^2\}V - mV\partial^{\alpha\dot\alpha}H_{\alpha\dot\alpha} + \frac{3}{2}m^2V^2\Big\}. \tag{A.2}$$

This is one of the two formulations for the massive superspin-3/2 multiplet constructed in [10].

The theory (2.15) also admits a dual formulation. Let us consider the following "first-order" action

$$S_{\mathrm{Aux}} = \int d^8z \Big\{ H^{\alpha\dot\alpha}\Box\Big(\frac{1}{2}\Pi^T_{\frac{3}{2}} - \Pi^T_{\frac{1}{2}}\Big)H_{\alpha\dot\alpha} - \frac{1}{2}m^2H^{\alpha\dot\alpha}H_{\alpha\dot\alpha} + \frac{1}{2}\mathcal{U}[D_\alpha,\overline{D}_{\dot\alpha}]H^{\alpha\dot\alpha} + \frac{3}{2}\mathcal{U}^2 \\ -6mV\Big(\mathcal{U} - D^\alpha\chi_\alpha - \overline{D}_{\dot\alpha}\overline{\chi}^{\dot\alpha}\Big)\Big\} + 3m^2\Big\{\int d^6z\,\chi^\alpha\chi_\alpha + \mathrm{c.c.}\Big\}, \tag{A.3}$$

in which $\mathcal{U}$ and V are real unconstrained superfields. Varying V gives the original action (2.15). On the other hand, we can eliminate the independent scalar $\mathcal{U}$ and chiral spinor χ_α variables using their equations of motion. With the aid of (2.8) this gives

$$S^{(\mathrm{IIB})}[H,V] = \int d^8z \Big\{ H^{\alpha\dot\alpha}\Box\Big(\frac{1}{2}\Pi^T_{\frac{3}{2}} - \frac{1}{3}\Pi^L_0\Big)H_{\alpha\dot\alpha} - \frac{1}{2}m^2H^{\alpha\dot\alpha}H_{\alpha\dot\alpha} \\ +mV[D_\alpha,\overline{D}_{\dot\alpha}]H^{\alpha\dot\alpha} - 6m^2V^2\Big\} - 6\int d^6z\,W^\alpha W_\alpha\,, \tag{A.4}$$

where $W_\alpha = -\frac{1}{4}\overline{D}^2 D_\alpha V$ is the vector multiplet field strength. The theory with action (A.4) was constructed in [14].

Finally, to construct a dual formulation for the theory (2.16), let us introduce the "first-order" action

$$S_{\rm Aux} = \int d^8z\, \Big\{ H^{\alpha\dot\alpha}\Box\Big(\frac{1}{2}\Pi^T_{\frac{3}{2}} + \frac{1}{3}\Pi^L_{\frac{1}{2}}\Big)H_{\alpha\dot\alpha} - \frac{1}{2}m^2 H^{\alpha\dot\alpha}H_{\alpha\dot\alpha} + \mathcal{U}\partial_{\alpha\dot\alpha}H^{\alpha\dot\alpha} + \frac{3}{2}\mathcal{U}^2$$
$$+3mV\Big(\mathcal{U} - D^\alpha\chi_\alpha - \overline{D}_{\dot\alpha}\overline{\chi}^{\dot\alpha}\Big)\Big\} - 9m^2\Big\{\int d^6z\,\chi^\alpha\chi_\alpha + {\rm c.c.}\Big\}, \quad \text{(A.5)}$$

in which $\mathcal{U}$ and V are real unconstrained superfields. Varying V gives the original action (2.16). On the other hand, we can eliminate the independent real scalar $\mathcal{U}$ and chiral spinor χ_α using their equations of motion. With the aid of (2.7) this gives

$$S^{(\rm IIIB)}[H,V] = \int d^8z\, \Big\{ H^{\alpha\dot\alpha}\Box\Big(\frac{1}{2}\Pi^T_{\frac{3}{2}} - \frac{1}{3}\Pi^L_0\Big)H_{\alpha\dot\alpha} - \frac{1}{2}m^2 H^{\alpha\dot\alpha}H_{\alpha\dot\alpha}$$
$$-mV\partial_{\alpha\dot\alpha}H^{\alpha\dot\alpha} - \frac{3}{2}m^2V^2\Big\} + \frac{1}{2}\int d^6z\, W^\alpha W_\alpha\,, \quad \text{(A.6)}$$

with a vector multiplet field strength W_α. This is one of the two formulations for the massive superspin-3/2 multiplet constructed in [10].

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